

Comments on Baryon Melting in Quark Gluon Plasma with gluon condensation

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Abstract

We consider a black hole solution with a non-trivial dilaton from IIB super gravity which is expected to describe a strongly coupled hot gauge plasma with non-vanishing gluon condensation present. We construct a rotating and moving baryon to probe the screening and phases of the plasma. Melting of the baryons in hot plasma in this background had been studied previously, however, we show that baryons melt much lower temperature than has been suggested previously.

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I. INTRODUCTION

The gauge/string duality [1] has generated much interest in investigating quantum chromodynamics (QCD) at finite temperature. Interesting quantities of hot QCD plasma, such as η/s , meson and baryon screening length (L_s), jet quenching parameter \hat{q} have been obtained by gravity calculation, while they are hard to compute in lattice. The data from heavy ion collisions at RHIC shows that, above a critical temperature $T_c \sim 170\text{MeV}$ (deconfinement transition), the QCD plasma is like a strongly coupled liquid [2] with approximately ideal hydrodynamics. On the other hand, lattice results indicate that, the thermodynamics of QCD plasma is scale invariant upto $\sim 2T_c$ [3]. Although these points provide plausibility for describing QCD plasma by a super Yang-Mill theory with a gravity dual with conformal invariance, to capture the behavior of real QCD, one needs to extend the gauge/gravity duality to nonconformal cases and there have been much efforts [4, 5] along this direction. It is remarkable that the screening length and jet quenching parameter can be affected by the nonconformality by $20 \sim 30\%$ [6].

One of the problem in holographic QCD is that the temperature dependence of the baryons are usually suppressed in holographic QCD, although it has been known that heavy quark bound states can survive in quark gluon plasma [7] in real QCD. In fact, for the most of the background with regular horizon, the finite energy configuration of the baryon vertex does not exist. However, in [8], it was found that in the system of non-extremal $D3/D_{-1}$, closed baryon vertex operator exists even at finite temperature. Therefore this background is ideal to study the temperature dependence of various quantities around the critical temperature. Especially interesting one is the baryon melting temperature and the associated phases.

Baryons in hot plasma in this background had been studied previously [9, 10, 11, 12, 13]. In [9, 10], one baryon was considered as a point vertex with hanging strings. It was shown that the velocity dependence of screening distance goes like $L_s \sim (1 - v^2)^{1/4}$. Baryons which have radius larger than L_s will dissociate. Since $L_s \sim 1/T$, for any fixed baryon radius R , there is a critical temperature $T_c \sim 1/R$ above which baryons will dissociate. In [11], the same analysis was made by taking into account the shape of baryon. In [12], we made screening related analysis using the non-conformal background found in [8].

Recently, the authors of [13] suggested another melting temperature T_m above which no compact D5 vertex exists by studying the DBI action of D5 vertex. In this paper, we study thermodynamics of this model in the presence of baryon vertex operators and show that the baryons melt at much lower temperature than it was suggested in [13] by comparing the free energy of baryon configuration and that of free string configuration. We also examine the

phases of a baryon in various motions like rotating and moving configurations.

II. D3/D-INSTANTON BACKGROUND AND GLUON CONDENSATION

The gravity theory dual to the thermal four-dimensional gauge theory is a solution of 10D Type-IIB supergravity under the Freund-Rubin ansatz for self-dual five form field strength [8, 14, 15]. In string frame, the solution can be written as follow

$$e^{-\frac{1}{2}\phi}ds_{10}^2 = -\frac{r^2}{r_+^2}\left(1 - \frac{r_0^4}{r^4}\right)dt^2 + \frac{r^2}{r_+^2}dx_idx^i + \frac{1}{1 - \frac{r_0^4}{r^4}}\frac{r_+^2}{r^2}dr^2 + r_+^2d\Omega_5^2, \quad (1)$$

with a dilaton and an axion

$$e^\phi = 1 + \frac{q}{r_0^4} \log \frac{1}{1 - \frac{r_0^4}{r^4}}, \quad \chi = -e^{-\phi} + \chi_0, \quad (2)$$

where $i = 1, 2, 3$ and q is gauge fields condensate parameter. ϕ and χ denote the dilaton and the axion respectively. This metric includes an AdS black hole times a five-dimensional sphere, the dilaton and axion depending on r . r_+ is the curvature radius of the AdS metric, r is the coordinate of the fifth dimension of AdS_5 and r_0 is the position of black hole horizon. The temperature of the gauge theory is given by Hawking temperature of the black hole, $T = \frac{r_0}{\pi r_+^2}$. By duality, the gauge theory parameters N_c and λ (t'Hooft coupling) are given by $\sqrt{\lambda} = \frac{r_+^2}{\alpha'}, \frac{\lambda}{N_c} = g_{YM}^2 = 4\pi g_s$, where $\frac{1}{2\pi\alpha'}$ is string tension and g_s is the string coupling constant. The self-dual Ramond-Ramond field strength is

$$F_{(5)} = dC_{(4)} = 4r_+^4\Omega_5d\theta_1 \wedge \dots \wedge d\theta_5 - 4\frac{r_+^3}{r^4}dt \wedge dx_1 \wedge dx_2 \wedge dx_3 \wedge dr, \quad (3)$$

where $\Omega_5 = \sin^4\theta_1 \sin^3\theta_2 \sin^2\theta_3 \sin\theta_4$.

The baryon construction in gravity involves N_c fundamental strings with the same orientation, beginning at the heavy quarks on the flavor brane and ending on the baryon vertex in the interior of bulk geometry [16], which is a D5 brane wrapped on the S^5 ($AdS_5 \times S^5$ background). [24] The D5 brane carries a radial U(1) flux and wraps the S^5 with radial extension. The action of D5 brane includes DBI action plus Chern-Simons action, given by

$$S_{D5} = -T_5 \int d^6\sigma e^{-\phi} \sqrt{-\det(g_{ab} + 2\pi\alpha' F_{ab})} + T_5 2\pi\alpha' \int A_{(1)} \wedge \mathcal{P}(F_{(5)}), \quad (4)$$

where the 6D world volume induced metric $g_{ab} = \partial_a X^\mu \partial_b X^\nu G_{\mu\nu}$, and the pull back of five form $\mathcal{P}(F_{(5)}) = \partial_{a_1} X^{\mu_1} \dots \partial_{a_4} X^{\mu_5} F_{\mu_1 \dots \mu_5}$. The D5 brane tension $T_5 = \frac{1}{g_s(2\pi)^5 l_s^6}$, and the world volume

field strength of U(1) flux $F_{(2)} = dA_{(1)}$. The Chern-Simons term endows D5 brane with U(1) charge. By the following consistent ansatz that describes the embedding D5 brane

$$\begin{aligned}\tau &= t, & \sigma_1 &= \theta, & \sigma_2 &= \theta_2, \dots \sigma_5 = \theta_5, \\ r &= r(\theta), & x &= x(\theta),\end{aligned}\tag{5}$$

we see only SO(5) symmetric configurations of D5 brane which stand for baryons in 4D real spacetime (t, \vec{x}) are considered, and the embedding function can be determined by $r(\theta)$ and $x(\theta)$. The gauge field on D5 can also be written as $A_t(\theta)$ for symmetry. The action of D5 brane is given by

$$S = T_5 \Omega_4 r_+^4 \int dt d\theta \sin^4 \theta \left[4A_t - e^{\frac{\phi}{2}} \times \sqrt{(1 - r_0^4 r^{-4})r^2 + r'^2 + (1 - r_0^4 r^{-4})r^4 r_+^{-4} x'^2 - F_{\theta t}^2} \right], \tag{6}$$

where $\Omega_4 = 8\pi^2/3$ is the volume of unit four sphere. To obtain the configuration of D5 brane, we should solve the gauge field at first. The equation of motion turns to be

$$\partial_\theta D = -4 \sin^4 \theta. \tag{7}$$

The solution to the above equation is

$$\begin{aligned}D(\nu, \theta) &= \frac{3}{2}(\sin \theta \cos \theta - \theta + \nu \pi) + \sin^3 \theta \cos \theta, \\ 0 &\leq \nu = \frac{k}{N_c} \leq 1,\end{aligned}\tag{8}$$

where k denotes the number of Born-Infeld strings emerging from south pole of S^5 . More details about this explanation can be found in [17]. To eliminate the gauge field in favor of D , we shall transform the original Lagrangian to obtain an energy functional of the embedding function as follow

$$\mathcal{H} = \tilde{T} \int d\theta e^{\frac{\phi}{2}} \sqrt{\left(1 - \frac{r_0^4}{r^4}\right)r^2 + r'^2 + \left(1 - \frac{r_0^4}{r^4}\right)\frac{r^4}{r_+^4}x'^2} \times \sqrt{D^2 + \sin^8 \theta}. \tag{9}$$

where $\tilde{T} = T_5 \Omega_4 r_+^4$. In order to find the configuration of D5 brane, one must extremize \mathcal{H} , with respect to $r(\theta)$ and $x(\theta)$ respectively. A closed solution of D5 is identified to be a physical baryon vertex. More discussion about these solutions can be found in [18]. By solving equation of motion for $r(\theta)$ and $x(\theta)$ in (9), we can find different kinds of solutions for baryon vertex.

A. Baryon vertex solutions

Note that point vertexes in real spacetime corresponds to $x'(\theta) = 0$. If $q = 0$, the gravity theory is the usual AdS black hole. In that case, vertex D5 brane with a DBI+CS action can

not have a closed solution. Here, choosing $q > 0$ (or some critical value) to keep D5 solutions closed. The solutions are independent on r_+ , if $x'(\theta) = 0$. Only two parameters q and r_0 determine behaviors of solutions. When we choose suitable parameters $q = 2$ and, r_0 between 0.1 and 0.689, the vertex brane solutions can have four typical behaviors according to the initial value $r(0)$ [18]. They correspond to four types of configurations of a baryon. From these solutions, we see that there is always a singularity in $r_e = r(\pi)$, if we give initial conditions $r'(0) = 0, r(0) = C$, where C is a constant.

B. Force balance condition

Adding fundamental strings can help to eliminate this singularity and keep charge conserved. For simplicity, consider that N_c fundamental strings all attach the north pole of S^5 , which means $\nu = 0$. N_c static quarks are arranged on a circle in (x_1, x_2) space, whose coordinates can also be written as (ρ, α) . By the following consistent ansatz that describes the embedding fundamental strings $\tau = t, \quad \sigma = r, \quad \rho = \rho(r)$, we write the string action $S_F = \frac{1}{2\pi\alpha'} \int dt dr \mathcal{L}_F$. To eliminate the singularity of cusp of D5 brane at r_e , one needs force balance conditions. One force balance condition in ρ direction is satisfied for central symmetry. Another force balance condition in r direction is given by

$$N_c \left\{ \mathcal{L}_F - \rho' \frac{\partial \mathcal{L}_F}{\partial \rho'} \right\} \bigg|_{r_e} = 2\pi\alpha' \frac{\partial \mathcal{H}}{\partial r_e}. \quad (10)$$

The left hand of equation (10) is up force of string and the right hand is down force of brane. The balance point is the singularity of vertex solution.

III. PHASES OF BARYON IN VARIOUS MOTION

A. Baryon in motion and the Binding energy

A baryon is consist of compact D5 brane and N_c strings coming out of the north pole of it. The latter is considered as quarks and we shall consider such quarks moving in medium and rotating in a plane, corresponding to boosted and high spin hadron state respectively [25]. The medium wind will effect the vertex brane and fundamental strings in the same time, and the vertex brane can not feel the rotating effect, because it is a central point in the rotation plane. We consider quarks moving in x_3 direction and rotating in (ρ, α) plane. For simplicity, we shall stand in the rest frame of the baryon configuration. The metric (1) can be boosted such that it describes a gauge plasma moving with a wind velocity v in the negative x_3 -direction. The

boosted metric is given by

$$e^{-\frac{1}{2}\phi}ds_{10}^2 = -Adt^2 + 2Bdtdx_3 + Cdx_3^2 + \frac{r^2}{r_+^2}(d\rho^2 + \rho^2 d\alpha^2) + \frac{r_+^2}{r^2} \frac{1}{f(r)} dr^2 + r_+^2 d\Omega_5^2 \quad (11)$$

where

$$A = \frac{r^2}{r_+^2} \left(1 - \frac{r_1^4}{r^4}\right), B = \frac{r_1^2 r_2^2}{r^2 r_+^2}, C = \frac{r^2}{r_+^2} \left(1 + \frac{r_2^4}{r^4}\right) \quad (12)$$

with

$$\begin{aligned} r_1^4 &= r_0^4 \cosh^2 \eta, \quad r_2^4 = r_0^4 \sinh^2 \eta, \\ v &= -\tanh \eta, \quad f = 1 - \frac{r_0^4}{r^4}. \end{aligned} \quad (13)$$

In the boosted metric, baryon configuration will depend on η . Both vertex brane and fundamental string solutions will be different from the original ones with $\eta = 0$. First, we pay attention to vertex brane solutions. For point brane vertex, one notes that x^i is independent on θ . The D5 brane action in boosted metric is given by

$$\mathcal{H}_\eta = \tilde{T} \int d\theta e^{\frac{\phi}{2}} \sqrt{\left(r^2 + \frac{1}{f} r'^2\right) \left(1 - \frac{r_0^4 \cosh^2 \eta}{r^4}\right)} \times \sqrt{D^2 + \sin^8 \theta}. \quad (14)$$

To obtain rotating fundamental string configuration, we give the following consistent ansatz of embedding function

$$\tau = t, \quad \sigma = r, \quad \alpha = \omega t, \quad \rho = \rho(r). \quad (15)$$

Facing the wind in x_3 direction, quarks arranged on the circle in $x_1 - x_2$ plane will keep staying in $x_1 - x_2$ plane and stand on a circle, because they all have the same force. Then the rotating string action in the boosted metric can be written as

$$\tilde{S}_F = \frac{\mathcal{T}}{2\pi\alpha'} \int_{r_e}^{r_\Lambda} dr \tilde{\mathcal{L}}_F, \quad (16)$$

where the Lagrangian

$$\tilde{\mathcal{L}}_F = e^{\frac{\phi}{2}} \sqrt{\left(1 - \frac{r_0^4 \cosh^2 \eta}{r^4} - \rho^2 \omega^2\right) \left(\frac{1}{f} + \frac{r^4}{r_+^4} \rho'^2\right)}. \quad (17)$$

To solve the equation of motion of strings, we need two initial conditions. One is known by $\rho(r_e) = 0$ for symmetry, and the other must be calculated by the force balance condition (10). To get the baryon radius in the boundary, we define $L_q = \int_{r_e}^{r_\Lambda} \rho'(r) dr$. For $\eta > 0, \omega = 0$, string Lagrangian contains no ρ and one can solve ρ' from equation of motion of ρ and express baryon radius L_q in terms of r_e and η by

$$L_q = \int_{r_e}^{r_\Lambda} dr \frac{K(r_e, \eta) r_+^4 (r^4 - r_0^4)^{-1/2}}{\sqrt{e^\phi (r^4 - r_0^4 \cosh^2 \eta) - K^2(r_e, \eta) r_+^4}} \quad (18)$$

where $K(r_e, \eta)$ is constant coming from the equation of motion of $\rho(r)$, $\partial_{\rho'} \tilde{\mathcal{L}}_F = K$, determined by the force balance condition

$$K(r_e, \eta) = \frac{\sqrt{(r_e^4 - r_0^4 \cos^2 \eta) e^{\phi(r_e)}}}{r_+^2 \sqrt{1 + \theta'^{-2} r_e^{-2} f^{-1}}} . \quad (19)$$

For $\eta > 0, \omega > 0$, equation of motion of ρ is difficult to solve analytically. One must search for the numerical result. Screening length is defined as the maximum of L_q while we change r_e .

The total baryon energy is given by sum of the energy of N_c strings and that of the baryon vertex :

$$E_{total} = N_c E_{string} + E_{D5}, \quad (20)$$

where the masses of string and vertex brane are given by

$$E_{string} = \omega \frac{\partial \tilde{L}}{\partial \omega} - \tilde{L}, \quad E_{D5} = \mathcal{H}_\eta, \quad (21)$$

Where $\tilde{L} = \frac{1}{2\pi\alpha'} \int_{r_e}^{r_\Lambda} dr \tilde{\mathcal{L}}_F$ is the string Lagrangian. We define the effective baryon binding energy as the difference between the total energy of a baryon and that of N_c quarks rooted at the black hole horizon. Notice that compact D5 brane wrapping the horizon has no mass. If the radius of horizon is r_0 , and boundary is at $r = r_\Lambda$, the mass of deconfined quark is given by

$$E_q = \frac{1}{2\pi\alpha'} \int_{r_0}^{r_\Lambda} e^{\phi/2} dr \quad (22)$$

Then the binding energy is given by

$$E_I = E_{total} - N_c E_q. \quad (23)$$

Concretely, E_I is written by

$$\begin{aligned} E_I = & \frac{N_c}{2\pi\alpha'} \int_{r_e}^{r_\Lambda} dr \frac{e^\phi}{\tilde{\mathcal{L}}_F} \left(\frac{1}{f} + \frac{r^4 \rho'^2}{r_+^4} \right) \left(1 - \frac{r_0^4 \cosh^2 \eta}{r^4} \right) \\ & + \mathcal{H}_\eta - \frac{N_c}{2\pi\alpha'} \int_{r_0}^{r_\Lambda} e^{\phi/2} dr. \end{aligned} \quad (24)$$

B. Phases of baryon

Our gravity background is an AdS black hole modified by a dilaton and an axion. In the vacuum $T = 0$, the dilaton controls the effective coupling constant of QCD, and the axion is dual to the QCD θ angel [5]. By the calculation of Wilson loop, one can find that, the effective potential exhibits confining nature within a temperature dependent screening length while for

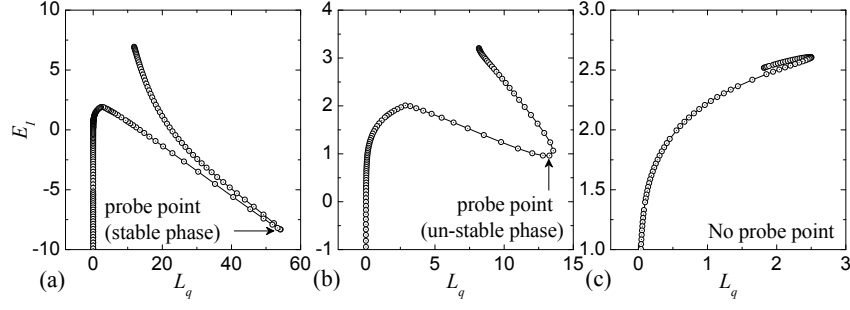


FIG. 1: a) Potential curves at low temperature. b) Potential curves at middle temperature. c) Potential curves at high temperature.

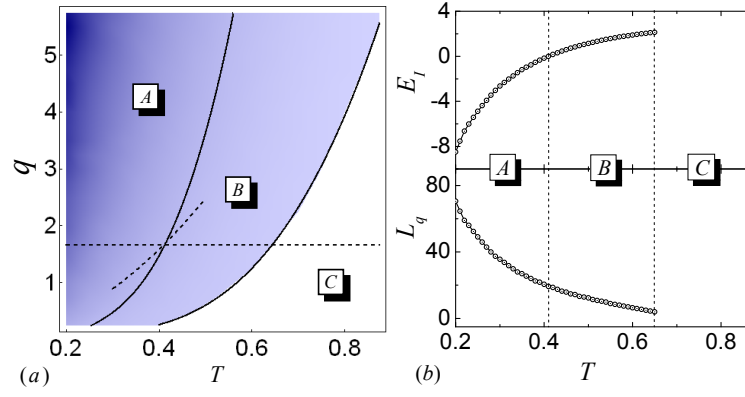


FIG. 2: LEFT: Stability of baryon: A: Region of stable baryon ($E_I < 0$). B: Un-stable baryon ($E_I > 0$). C: Region where no baryon configuration is allowed. The horizontal dash line is $q = q_c$ and the short dash line is used to determine $q_c = 1.67$. RIGHT: T dependence of bind energy and the baryon size, with $q = q_c$.

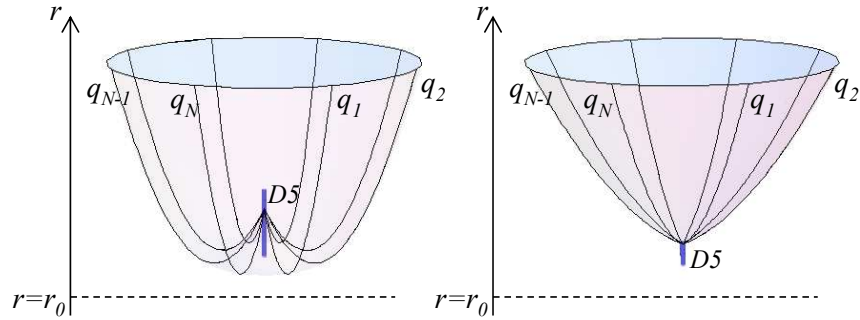


FIG. 3: LEFT: Baryon configuration at low temperature, which corresponds to the case there is a minimal potential. RIGHT: Baryon configuration at high temperature, which exhibits monotonic potential.

large separation of quark and anti quark experience confining potential. If we introduce a hard wall in IR regime by hand, we could introduce (gluon) confining background IR [8]. However, such confinement transition by geometry changing is relevant to the gluon dynamics and it is not directly related to that of quarks and baryons. Since our interest is the dynamics of the quarks/baryons, we do not introduce a hard wall and therefore we do not have Hawking Page transition.

1. Heavy probe baryon configuration and Critical temperature

We shall focus on the effective potential between quarks of baryon, which was derived in (24). To make this part clear, we need some background knowledge. As is well known, in pure gauge theory, to understand confinement, we are particularly interested in the effective potential of gauge interaction between external fundamental probes. In thermal gauge theory and ones with flavors, the effective potential comes from more complex sources, where we should take account of finite temperature fluctuations and some flavor dynamics. Here we study the finite temperature case but leave the flavor dynamics to the future work. In particular, we focus on lower temperature backgrounds than the usual backgrounds where the potential curves (E_I vs L_q) exhibit monotonic behaviors (we only take the lower branch of potential curves), which are shown in (c) in Fig.1. In thermal plasma, we can think temperature T is a typical scale. Now we focus on baryons with radius L_q larger than a cutoff L_c , where $L_c \sim \frac{1}{T}$. Note that the potential curve reflects the interaction of quarks of baryon at different separations. As T decreases, we obtain the potential curves typically like (b) in Fig.1. We consider that baryons in the bottom of the curves are more stable. Ignoring the infinitely negative piece of the left part of curves in (b) in Fig.1 since the cutoff, we observe that there appears a minimal point of potential for each curve. In the following sections, we focus on baryon configurations at this point. We call this point “probe point”. Since E_I can also be defined as binding energy of baryon, for negative E_I , baryon is more stable than N_c free quarks, therefore quarks will bound into hadrons. Thus, we consider the “probe point” in (a) in Fig.1 is a really stable baryon. And critical temperature T_c can be defined by “probe point” with $E_I = 0$. [26]

In Fig.1, we plotted potential curves VS baryon size (E_I vs L_q). Notice that there exist three typical potential curves for low temperature (a), middle temperature (b) and high temperature (c). Above T_m ($T_m > T_c$), there is no stable baryon and no “probe point”. So previously T_m was considered as the melting temperature of baryons. However, here we see that if temperature is higher than the T_c , quark phase is the preferred one and therefore baryon should melt before

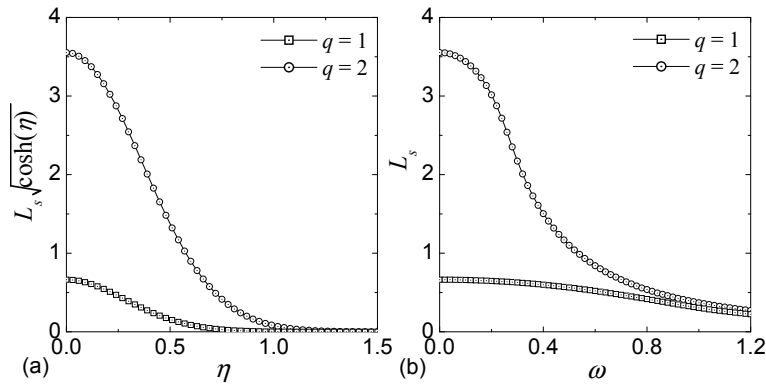


FIG. 4: LEFT: velocity dependence of screening length. RIGHT: Rotation dependence of screening length.

we reach the T_m . We plotted the size and effective potential of “probe points” states covering a wide temperature and condensation region in Fig.2 (LEFT). We also plotted the baryon potential and size depending on T in Fig.2 (RIGHT). The reader may be puzzled about where the “probe points” come from. The physical reason is that at low temperature, the projection line of vertex brane solution is not monotone increasing in r direction as θ increases from 0 to π . The configuration of the whole baryon is described by left one in Fig.3, which can be compared with the right normal one at higher temperature. Note that, in order to calculate the quark separation and effective potential of baryon for the left one in Fig.3, one should integrate two parts of one whole fundamental string, that is $r_e \rightarrow r_{min}$ and $r_{min} \rightarrow r_{max}$.

2. Screening effect at high temperature

In higher temperature background, we have no stable baryon by potential study. However, we can obtain the screening effect, which provides the screening length of baryon (even unstable). According to the screening analysis in section III.A, for the boosted heavy baryon and high spin heavy baryon, we calculate the η and ω dependence of the screening length at $r_0=0.7$. As shown in Fig.4 [12], gluon condensation makes the screening length larger than before. Baryon screening length obeys $L_s \sim (1 - v^2)^{1/4}$ at large velocity.

IV. CONCLUSION

In this letter, we probe the hot nonconformal QCD plasma by a heavy baryon, and find that the baryons actually melt before it reach the “melting temperature T_m ” beyond which baryon configuration does not exist. We determine the critical temperature T_c , the quark state

is energetically more favorable than the baryon state. T_c is lower than T_m .

We also calculated the temperature dependence of the free energies for each phase and the temperature dependence of baryon size. In the high temperature region, we calculate boost velocity η and spin dependence of screening length.

There are different approaches to the baryon in medium advocated in [22]. The idea is that flavor probe brane is deformed by attached strings which are emanating from the baryon vertex. Since the fundamental string is more expensive than the deformed probe brane or baryon vertex, two branes are in contact with force balancing condition. It would be very interesting to study the phase transition point T_c in this picture. Study in this direction is currently under progress.

One can also study the phase transition by considering the fermionic charges [23] more explicitly.

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- [24] Generally, N_c quarks are allowed to be placed at arbitrary positions in \vec{x} space on the boundary. Note that these quarks are heavier than component quarks of light mesons, and the N_c quarks

- bound states can not easily be considered as an effective field on the boundary (fluctuations of flavor brane in the picture with flavor) [10].
- [25] Actually, the general shape of multi quarks is a sphere in 3D space for largest symmetry, rather than circular.
- [26] We set $q = q_c$ to describe our background in the middle temperature region. We compare our background with that in work [19], and determine $q_c = 1.67$ in Fig.2.